6.3  (a) Since the model is linear in the parameters, it is a linear regression model.

(b) Define $Y^* = (1/Y)$ and $X^* = (1/X)$ and do an OLS regression of $Y^*$ on $X^*$.

(c) As $X$ tends to infinity, $Y$ tends to $(1/\beta_i)$.

(d) Perhaps this model may be appropriate to explain low consumption of a commodity when income is large, such as an inferior good.

6.10 As discussed in Sec. 6.7 of the text, for most commodities the Engel model depicted in Fig. 6.6(c) seems appropriate. Therefore, the second model given in the exercise may be the choice.

6.14 The regression results are as follows:

$$\log \left( \frac{Y}{L} \right) = -0.4526 + 1.3338 \log W$$

\[se = (1.3515) (0.4470) \quad r^2 = 0.4070\]

To test the null hypothesis, use the $t$ test as follows:

$$t = \frac{1.3338 - 1}{0.4470} = 0.7468$$

For 13 df, the 5% (two-tail) critical $t$ value is 2.16. Therefore, do not reject the hypothesis that the true elasticity of substitution between capital and labor is 1.

6.16 The regression results are:

$$\hat{Y}_i^* = 0.9892X_i^*$$

\[se = (0.0388) \quad r^2 = 0.9789\]

A one standard deviation increase in the GDP deflator for imports results in a 0.9892 standard deviation increase in the GDP deflator for domestic goods, on average. Note that this result is comparable to the one given in the preceding problem when one notes the relationship between slope coefficients of the standardized and non-standardized regressions. As shown in Eq. (6.3.8) in the text,

$$\beta_2^* = \beta_2 \left( \frac{S_x}{S_y} \right), \text{ where } * \text{ denotes slope from the standardized regression.}$$

In the previous problem we found $\hat{\beta}_2 = 0.5340$. $S_y$ and $S_x$ are given as 346 and 641, respectively. Therefore,

$$\beta_2 \left( \frac{S_x}{S_y} \right) = 0.5340 \left( \frac{641}{346} \right) = 0.9892 = \hat{\beta}_2.$$  

7.2 Using the formulas given in the text, the regression results are as follows:

$$\hat{Y}_i = 53.1612 + 0.727X_{2i} + 2.736X_{3i}$$

\[se = (0.049) (0.849) R^2 = 0.9988; \quad \overline{R^2} = 0.9986\]
7.9 The slope coefficients in the double-log models give direct estimates of the (constant) elasticity of the left-hand side variable with respect to the right hand side variable. Here:
\[
\frac{\partial \ln Y}{\partial \ln X_2} = \frac{\partial Y / Y}{\partial X_2 / X_2} = \beta_2, \text{ and} \\
\frac{\partial \ln Y}{\partial \ln X_3} = \frac{\partial Y / Y}{\partial X_3 / X_3} = \beta_3
\]

7.12 (a) Rewrite Model B as:
\[
Y_i = \beta_1 + (1 + \beta_2) X_{2i} + \beta_3 X_{3i} + u_i \\
= \beta_1 + \beta'_2 X_{2i} + \beta_3 X_{3i} + u_i, \text{ where } \beta'_2 = (1 + \beta_2)
\]
Therefore, the two models are similar. Yes, the intercepts in the models are the same.

(b) The OLS estimates of the slope coefficient of $X_3$ in the two models will be the same.

(c) $\beta'_2 = (1 + \beta_2) = \alpha_2$

(d) No, because the regressands in the two models are different.

7.22 The results of fitting the Cobb-Douglas production function, obtained from Eviews3 are as follows:

**Dependent Variable: LOG(OUTPUT)**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>-11.93660</td>
<td>3.211064</td>
<td>-3.717335</td>
<td>0.0011</td>
</tr>
<tr>
<td>LOG(LABOR)</td>
<td>2.328402</td>
<td>0.599490</td>
<td>3.883972</td>
<td>0.0007</td>
</tr>
<tr>
<td>LOG(CAPITAL)</td>
<td>0.139810</td>
<td>0.165391</td>
<td>0.845330</td>
<td>0.4063</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.971395</td>
<td>Mean dependent var</td>
<td>4.493912</td>
<td></td>
</tr>
<tr>
<td>Adjusted R-squared</td>
<td>0.969011</td>
<td>S.D. dependent var</td>
<td>0.461432</td>
<td></td>
</tr>
<tr>
<td>S.E. of regression</td>
<td>0.081229</td>
<td>Akaike info criterion</td>
<td>-2.078645</td>
<td></td>
</tr>
<tr>
<td>Sum squared resid</td>
<td>0.158356</td>
<td>Schwarz criterion</td>
<td>-1.934663</td>
<td></td>
</tr>
<tr>
<td>Log likelihood</td>
<td>31.06171</td>
<td>F-statistic</td>
<td>407.5017</td>
<td></td>
</tr>
<tr>
<td>Durbin-Watson stat</td>
<td>0.373792</td>
<td>Prob(F-statistic)</td>
<td>0.000000</td>
<td></td>
</tr>
</tbody>
</table>

(a) The estimated output/labor and output/capital elasticities are positive, as one would expect. But as we will see in the next chapter, the results do not make economic sense in that the capital input has no bearing on output, which, if true, would be very surprising. As we will see, perhaps collinearity may be the problem with the data.

(b) The regression results are as follows:
Dependent Variable: LOG(PRODUCTIVITY)

Date: 07/29/00   Time: 18:11
Sample: 1961 1987
Included observations: 27

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>-1.155956</td>
<td>0.074217</td>
<td>-15.57533</td>
<td>0.0000</td>
</tr>
<tr>
<td>LOG(CLRRATIO)</td>
<td>0.680756</td>
<td>0.044535</td>
<td>15.28571</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

R-squared 0.903345  Mean dependent var -2.254332
Adjusted R-squared 0.899479  S.D. dependent var 0.304336
S.E. of regression 0.096490  Akaike info criterion -1.767569
Sum squared resid 0.232758  Schwarz criterion -1.671581
Log likelihood 25.86218  F-statistic 233.6528
Durbin-Watson stat 0.263803  Prob(F-statistic) 0.000000

The elasticity of output/labor ratio (i.e., labor productivity) with respect to capital/labor ratio is about 0.68, meaning that if the latter increases by 1%, labor productivity, on average, goes up by about 0.68%. A key characteristic of developed economies is a relatively high capital/labor ratio.